Name: ..……………………………………………………………………………….

Student Number: ……………………………………………………………………..

Test 6 on WPPH16001.2020-2021
"Electricity and Magnetism"

Content: **5 questions**

Monday 14 June 2021; **online**, 9:00-12:00

- Write your full name and student number on **each** page you use
- Read the questions carefully. Read them one more time after having answered them to ensure you have answered exactly what you were asked for.
- Compose your answers is such a way that it is well indicated which (sub)question they address
- Upload the answer to each question as a **separate pdf file**
- Do not use a red pen (it's used for grading)
- Griffiths' textbook, lecture notes and **your** tutorial notes are allowed. The internet, mobile phones, consulting, requests for consultancy and other teamwork are not allowed and considered as cheating

Exam drafted by (name first examiner) Maxim S. Pchenitchnikov

Exam reviewed by (name second examiner) Steven Hoekstra

For administrative purposes; do NOT fill the table

The weighting of the questions:

	Maximum points	Averaged points scored
Question 1	11	
Question 2	15	
Question 3	13	
Question 4	12	
Question 5	12 + 2 *bonus*	
Total	**63**	

Grade = 1 + 9 x (score/max score)

Averaged grade:

Question 1 (11 points)

You shine a laser beam from underwater ($n_1 = 1.3$) through the flat water surface into the air ($n_2 = 1$). You are interested in the dependence of the reflection coefficient R at the water-air interface on the angle of incidence, for both p- and s- polarizations.

1. Calculate R for normal incidence. (1 point)

2. Is there an angle of incidence at which the reflection is nullified? For which polarization? Explain your answer with equations and provide the numerical value of the angle (3 points)

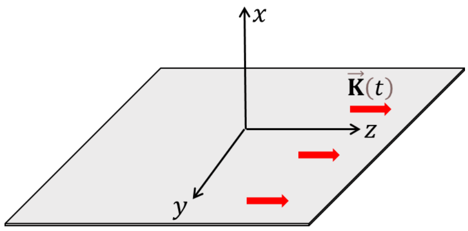
3. At which angle(s) does the reflection become 100%? For which polarizations? (4 points)

4. Based on the results of 1-3, sketch the dependence of the reflection coefficient as a function of the incident angle in the range 0-90 degree. (3 points)

Question 2. (15 points)

A thin metal electrically-neutral sheet, lying in the yz -plane, carries the uniform time-dependent current

$$\vec{\mathbf{K}}(t) = \begin{cases} 0, t \le 0 \\ K_0\, \hat{\mathbf{z}}, t > 0 \end{cases}$$



where K_0 is a constant.

1. Show that the vector potential at a height x above the plane is

$$\vec{\mathbf{A}}(x,t) = \frac{\mu_0 K_0 (ct - x)}{2}\hat{\mathbf{z}} \text{ (8 points)}$$

<u>Tip</u>. Don't forget of retardation!

2. Determine the electric field $\vec{\mathbf{E}}(x,t)$ at a height x above the plane. (3 points)

<u>Tip 1</u>. Don't forget that the field might be discontinuous; provide the values before and after the break point.

<u>Tip 2.</u> For writing up the final result, you might use the Heaviside step function

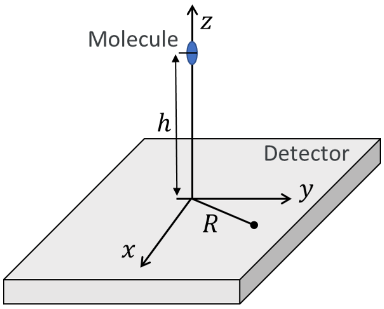
$$H(x) = \begin{cases} 0, x < 0 \\ 1, x \ge 0 \end{cases}$$

3. Determine the magnetic field $\vec{\mathbf{B}}(x,t)$ at a height x above the plane; see also the tips on $\vec{\mathbf{E}}$. (3 points)

4. Would you qualify these fields as an electromagnetic wave? Explain your answer. (1 point)

Question 3 (13 points)

You would like to study the radiation diagram of a small molecule of a size of ~1 nm. You detect the emitted radiation at the wavelength of $\lambda = 500$ nm with a two-dimensional detector (e.g., a photographic plate or a CCD matrix) which is positioned in the plane orthogonal to the direction in which the dipole moment of the molecule oscillates (see the figure for the experimental arrangement). The distance from the molecule to the detector is $h = 1$ mm.



1. Check if all approximations we made for deriving the parameters for the electric dipole radiation, hold for this arrangement. Provide the arguments (in numbers!) to support your answers. (3 points)

2. Show that the intensity of the radiation hitting the detector is

$$I(R) = \frac{3\langle P\rangle}{8\pi}\frac{R^2 h}{(R^2 + h^2)^{5/2}}$$

where R is the distance from the projection of the molecule onto the detector, and $\langle P\rangle$ is the total radiated power (measured is a separate experiment). (5 points)

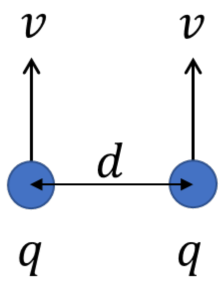
Tip. Don't forget that you need to calculate the power per unit area *of the detector*.

3. At which distance R_{Max} is the intensity maximal? (3 points)

4. Sketch the graph of dependence of detected intensity on the distance R from the projection of the dipole onto the detector. (2 points)

Question 4. (12 points)

Two identical charges q move along the parallel trajectories a distance d apart with speeds v (see the figure). We are interested in the force exerted upon the right charge due to the left charge, in the lab (i.e. the rest) system of coordinates.



1. Find the component of the Lorentz force $\vec{\mathbf{F}}_E$ due to electric field $\vec{\mathbf{E}}$ (2 points)

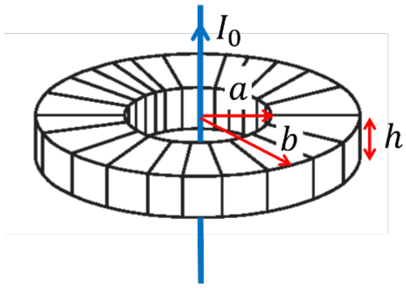
2. Find the component of the Lorentz force $\vec{\mathbf{F}}_B$ due to magnetic field $\vec{\mathbf{B}}$ (5 points)

3. Draw a diagram of the forces with both $\vec{\mathbf{F}}_E$ and $\vec{\mathbf{F}}_B$ in the figure. Make sure to include your coordinate axis for reference. (2 points)

4. Calculate the total Lorentz force, and take the limit $v \to c$ (2 points)

5. Now find the total force in the system of coordinates where charges are at rest. Did you get the same force as in #4? (1 point)

<u>Question 5. (12 points + 2 *bonus* points)</u>



The self-inductance of an ideal toroidal coil (see the figure) with rectangular cross-section (inner radius a, outer radius b, height h), that carries a total of N turns, was calculated as

$$L = \frac{\mu_0 N^2 h}{2\pi} ln\left(\frac{b}{a}\right)$$

by considering the flux of the magnetic field (see Example 7.11)

1. Calculate the self-inductance using the energy stored in the coil (do not reproduce Example 7.11 – it won't count). (4 points)

2. A straight wire runs along the axis of the toroidal coil. The wire carries an alternating current $I(t) = I_0 \cos \omega t$ with I_0 pointing *upward*. The coil is connected to a resistor with resistance R. In the quasistatic approximation, find the current, $I_R(t)$, in the resistor. (5 points)

3. Calculate the back emf in the coil, due to the current $I_R(t)$. (2 points)

4. Formulate (in words) what needs to be satisfied in order to justify the use of the quasistatic approximation here. (1 point)

5. *Bonus:* Quantify #4 by calculating the maximal frequency ω at which the quasistatic approximation still holds, if the size of the toroid is of the order of $a, b \sim 3$ cm (2 *bonus* points)

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June 8 2021